The Potential Fate of Local Model Building

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Abstract

We analyse local models at the point of E_8 in F-theory GUTs and identify exactly two models with potentially realistic properties concerning proton stability and a suitable pattern of quark and lepton masses. To this end we identify a matter parity at the local point. A globally consistent ultraviolet completion turns out to be problematic for both models. It is impossible to embed the models in a semilocal scheme via the spectral cover approach. This seems to severely limit the predictive power of local model building and to indicate that the full string theory might give us valuable hints for particle physics model building.

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1 Introduction

Exceptional groups might play a crucial role in the incorporation of grand unified theories (GUTs) in the framework of string theory. This is obvious in the $E_8 \times E_8$ heterotic theory [1], and more recently E_8 has been considered in F-theory [2] as well. Of course, E_8 is not an acceptable symmetry for a GUT in d=4 space time dimensions. The breakdown of E_8 and further properties of the d=4 theory depend crucially on the process of compactification. Such a picture has been analysed in the framework of the "heterotic braneworld" [3] where it led to the concept of "local grand unification"[1] [4–6]. The geography of localised matter fields and the nontrivial profile of gauge symmetries in compactified space allowed a successful incorporation of grand unification within string theory, thus providing a consistent

[1]Note that, in contrast to "local models", the concept of "local grand unification" refers to *global* models where the gauge symmetry exhibits a locally nontrivial profile in the compactified extra dimensions.

ultraviolet completion (for a review see [7]). F-theory leads to a qualitatively similar picture through intersecting branes and localised fields in extra dimensions. In contrast to the heterotic case, F-theory model building so far has generally relied on a bottom-up approach (see, however, [8]) analysing mostly the vicinity of special (local) points in compactified space, while ignoring the constraints from global consistency of the underlying theory (for a review see [9]).

The most attractive and predictive set-up concerns "The point of E_8" [10], where various branes cross at a point with local symmetry enhancement to E_8. This local picture has been analysed with respect to particle physics model building [11–18] with the goal to obtain the minimal supersymmetric extension of the standard model (MSSM). The present paper is an attempt to investigate the predictive power of such a local model and see which properties of particle physics could find a local explanation. In a second step we shall then apply constraints from global consistency and its implications for the local construction.

From all the properties of GUTs or the MSSM it is the superpotential that should find its explanation through the properties of the local model. The superpotential is relevant for Yukawa couplings (thus quark and lepton masses) as well as potential dangerous terms that might lead to fast proton decay. In the MSSM such terms are forbidden by a symmetry like matter parity P_M [19] (or generalisations thereof [20, 21]). We would therefore like to address the following question:

* Does the local model allow for proton stability with correct quark and lepton masses?

Correct Yukawa couplings should provide the top quark mass at the trilinear level and might explain other quark and lepton masses (if zero at the trilinear level) at higher order via a variant of the Froggatt–Nielsen mechanism [22].

Our analysis is performed in the framework of an $SU(5)$ GUT as defined in Ref. [15]. Higher unified groups like $SO(10)$ or E_6 will lead to more restrictive model building and will not be considered here. In Section 2 we shall present the setup of local models in detail. We identify the various curves that could support matter multiplets in $\mathbf{10}$ and $\overline{\mathbf{5}}$ representations of $SU(5)$, as well as $\mathbf{5}$ and $\overline{\mathbf{5}}$ representations for Higgs fields and singlets. We then display the general form of the superpotential that might be generated in such a scheme.

Section 3 analyses the prospects for realistic model building. We assume a trilinear coupling for the top quark and identify candidates for matter parity P_M. The scheme turns out to be quite restrictive: Only two candidates for P_M are allowed. We then identify the curves that carry the various matter and Higgs fields and check whether they allow for all quark and lepton masses and are consistent with P_M and proton stability. A scan of all possibilities then leads to two surviving models, exactly one for each of the two choices of P_M. This is a remarkable result: The local E_8 point is rather predictive.

It remains to be seen, however, whether the two local models can be incorporated within a globally consistent scheme. To construct the local models we had to make some assumptions about fluxes that determine the chirality of matter fields and split the Higgs multiplets. Such assumptions are restricted from global considerations. In a first step towards a global

realisation we consider a semilocal framework where information about the 8-dimensional (but not yet the full 12-dimensional) GUT surface is included. To perform the analysis we use the so-called spectral cover approach [23, 24] as the (so far) only available tool for this discussion (see, however, [25]). A summary of spectral cover results is given in Section 4 followed by a scan of the local models, with particular emphasis on the two successful models of Section 3. The result is quite striking: Both models are ruled out since there is no consistent extension to a global model. They fail already at the level of the semilocal completion. This suggests that realistic models of particle physics need to have some "nonlocal ingredients" that are not captured by the local point. Thus the predictions of local models might not be trustworthy in absence of a consistent global completion. A thorough discussion of F-theory "predictions" as well as conclusion and outlook will be given in Section 5.

2 Local Models, Operators and Matter Parity

F-theory GUT models [11, 12] are a generalisation of type IIB intersecting branes which allow for exceptional local symmetry groups, and thus in particular for a top quark Yukawa coupling, which requires a local E_6 enhancement. Similar to intersecting branes, one usually considers local models, in which one concentrates on the branes carrying the GUT symmetry, or curves and points within it. This "bottom-up" approach has the usual advantage that there is much more freedom in model building and one can ignore many of the problems of a global construction. The obvious disadvantage is that one does not know whether there is a global completion. Furthermore, some issues such as moduli stabilisation can only be tackled in a global model.

In Section 2.1 we will introduce the framework of local F-theory models. Since this is by now well-known, we will be rather brief, for a detailed review see e.g. [9]. After that, we introduce the relevant operators – Yukawa couplings and proton decay – and the matter parity we want to impose.

2.1 Local Models

Global F-theory models describe general vacua of type IIB string theory in terms of an elliptically fibred Calabi–Yau fourfold X, where seven-branes are indicated by the degeneration locus of the elliptic fibration. The general idea of local F-theory GUT models is to decouple the bulk of X and focus instead on a seven-brane on a submanifold S where the GUT symmetry, which will be $G_{\mathrm{GUT}} = SU(5)$ in our case, is localised. The intersections with other branes are visible as enhancements of the local symmetry group to $G_\Sigma \supset G_{\mathrm{GUT}}$ on curves Σ of complex codimension one. On these matter curves, there are localised hypermultiplets, the representation of which can be inferred from the decomposition of the adjoint of G_Σ. For $G_{\mathrm{GUT}} = SU(5)$, we get matter in the **5** and **10** representations from enhancements to $SU(6)$ and $SO(10)$, respectively.

Furthermore, the matter curves can intersect in points, and on these intersections the gauge group enhances even further to G_P. There will be localised Yukawa couplings from the cubic interaction of the adjoint of G_P. For an $SU(5)$ GUT, the up-type and down-type Yukawa couplings require pointwise enhancements to E_6 and $SO(12)$, respectively.

Since the surface S locally carries gauge groups larger than G_{GUT}, there is a different perspective: One can (at least locally) think of the worldvolume theory on S as a gauge theory with larger gauge group which is broken by a position-dependent vacuum expectation value (VEV) for an adjoint Higgs field. Since we want at least an E_6 enhancement, the largest possible gauge group is E_8, which contains $G_{\mathrm{GUT}} = SU(5)$ and its commutant, $E_8 \supset SU(5) \times SU(5)_\perp$. This is then broken to $SU(5) \times U(1)^4$ by the Higgs field. The extra $U(1)$'s correspond to the transverse branes and are generically broken in F-theory[2], but remain as global selection rules for the Lagrangean.

The matter curves are now the loci where certain components of the Higgs field vanish, such that the unbroken group is enhanced. To see what representations are localised where, we note the decomposition of the **248** of E_8:

$$\begin{aligned} E_8 &\longrightarrow SU(5) \times SU(5)_\perp \\ \mathbf{248} &\longrightarrow (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus (\mathbf{10},\mathbf{5}) \oplus \big(\overline{\mathbf{5}},\mathbf{10}\big) \oplus \big(\overline{\mathbf{10}},\overline{\mathbf{5}}\big) \oplus \big(\mathbf{5},\overline{\mathbf{10}}\big) \end{aligned} \tag{2.1}$$

We consider a diagonalisable Higgs field Φ and introduce a basis e_i, $i = 1, \ldots, 5$, for the **5** of $SU(5)_\perp$, such that $\Phi e_i = t_i e_i$. The Higgs eigenvalues t_i have to satisfy the tracelessness condition

$$\sum_{i=1}^{5} t_i = 0\,. \tag{2.2}$$

The **10** and the non-Cartan elements of the adjoint are spanned by the antisymmetric products $e_i \wedge e_j$ and by $e_i \wedge e_j^*$, respectively, where $i \neq j$. These elements are again eigenvectors of Φ with eigenvalues $t_i + t_j$ and $t_i - t_j$. The t_i vary over the GUT surface, and the matter is localised on their vanishing locus. From the decomposition (2.1) we can see how the representations of $SU(5)$ and $SU(5)_\perp$ are paired up, and focusing now on the representations of the unbroken $SU(5)$, we find the following equations for the matter curves:

$$\mathbf{10}\colon t_i = 0\,, \qquad \mathbf{5}\colon -(t_i + t_j) = 0\,, \qquad \mathbf{1}\colon \pm(t_i - t_j) = 0\,. \tag{2.3}$$

For the $\overline{\mathbf{10}}$ and $\overline{\mathbf{5}}$, there is an overall minus sign in the equation, because they correspond to conjugate representations of $SU(5)_\perp$.

The t_i determine the geometry of a deformed E_8 singularity in terms of the Tate model

$$y^2 = x^3 + b_5 xy + b_4 x^2 w + b_3 y w^2 + b_2 x w^3 + b_0 w^5\,. \tag{2.4}$$

[2]This is a global issue that cannot be analysed in a local model [26].

The GUT surface S is located at $w = 0$, and the coefficients b_k are the elementary symmetric polynomials in the t_i of degree k. We can rephrase the conditions (2.3) for the localisation of matter representations in terms of the b_k, which yields

$$\mathbf{10}: b_5 = 0\,, \qquad\qquad \mathbf{5}: b_3^2 b_4 - b_2 b_3 b_5 + b_0 b_5^2 = 0\,. \tag{2.5}$$

Note that the b_k do not fully specify the t_i, since the relation is nonlinear. In particular, this means there can be monodromies interchanging some of the t_i (since the b_i are symmetric polynomials), which amounts to identifying matter curves, and effectively reducing the commutant $SU(5)_\perp$ and the number of $U(1)$'s that remain.

If all matter curves meet at one single point, then all the $t_i = 0$, and thus all $b_k = 0$ except b_0. At this point the singularity is enhanced to E_8, as can be seen from Eq. (2.4), which becomes $y^2 = x^3 + w^5$. In the following we will assume that this is the case, and that all interactions come from this point of maximal enhancement. This has the advantage that the allowed interactions are determined purely by the quantum numbers, and there are no geometric suppression effects due to separations of Yukawa points. It has also been argued in [10] that this structure is favourable for incorporating proper masses and mixings in both the quark and lepton sector, including neutrino masses which we will ignore in this work.

If we had a global model at hand, we could determine the b_k and the matter curves, hence also the t_i. However, in the local approach we focus on the surface S and the point of E_8 enhancement. We will consider two degrees of locality: In the strictly *local* point of view, adopted in Section 3, we consider only the single point of E_8, such that we are dealing with a purely four-dimensional theory. This in particular implies that we can freely choose the monodromy group and the chirality of the zero modes on the matter curves (recall that matter comes in hypermultiplets, so that the chiral spectrum will be determined by some index theorem involving fluxes on the matter curves), and we will further assume that we can use a globally trivial, but locally nontrivial hypercharge flux to achieve doublet-triplet splitting for the Higgs **5**'s without introducing exotics on other curves. In Section 4 we will then consider a *semilocal* approach, in which we use spectral cover techniques to see whether this can actually be realised. At this level, we still assume a decoupled bulk, but take a more global look at the GUT surface S, i.e. we consider an eight-dimensional model. In particular, we realise the monodromy by a suitable choice of spectral cover, and find correlations between the homology classes of various matter curves. This implies that the restrictions of the hypercharge flux to the curves are correlated as well. Hence, it will turn out that the assumption of easy doublet-triplet splitting is not satisfied, which will lead to problems in the spectrum [15, 27].

2.2 The Good, the Bad, the Parity

Our aim in the following sections will be to find a $SU(5)$ GUT model, i.e. an assignment of fields to matter curves, which satisfies some basic phenomenological requirements. In particular, we demand masses for up- and down-type quarks and proton stability. To this

end we will look for a matter parity $P_M \subset SU(5)_\perp$ which forbids many operators leading to proton decay. We assume that all operators which are allowed by gauge symmetry and matter parity are generated with order one coefficients.

For the mass terms, some (good) operators of the form

$$\mathbf{1}_a \cdots \mathbf{1}_b \mathbf{5}_{H_u} \mathbf{10}_M \mathbf{10}_M \,, \qquad\qquad \mathbf{1}_a \cdots \mathbf{1}_b \overline{\mathbf{5}}_{H_d} \overline{\mathbf{5}}_M \mathbf{10}_M \tag{2.6}$$

should be allowed, so that VEVs for the singlets $\mathbf{1}_a$ lead to mass terms for all quarks and leptons. However, we require the $\mathbf{10}\,\mathbf{10}\,\mathbf{5}$ Yukawa coupling for the third generation at tree level to ensure a heavy top quark.

The relevant baryon and lepton number violating (bad) operators are [15]

$$\begin{aligned} W_{\not{B},\not{L}} = {} & \beta_i \overline{\mathbf{5}}^i_M \mathbf{5}_{H_u} + \lambda_{ijk} \overline{\mathbf{5}}^i_M \overline{\mathbf{5}}^j_M \mathbf{10}^i_M + W^1_{ijkl} \mathbf{10}^i_M \mathbf{10}^j_M \mathbf{10}^k_M \overline{\mathbf{5}}^l_M \\ & + W^2_{ijk} \mathbf{10}^i_M \mathbf{10}^j_M \mathbf{10}^k_M \overline{\mathbf{5}}_{H_d} + W^3_{ij} \overline{\mathbf{5}}^i_M \overline{\mathbf{5}}^j_M \mathbf{5}_{H_u} \mathbf{5}_{H_u} + W^4_i \overline{\mathbf{5}}^i_M \overline{\mathbf{5}}_{H_d} \mathbf{5}_{H_u} \mathbf{5}_{H_u} \end{aligned} \tag{2.7}$$

from the superpotential and

$$K_{\not{B},\not{L}} = K^1_{ijk} \mathbf{10}^i_M \mathbf{10}^j_M \mathbf{5}^k_M + K^2_i \overline{\mathbf{5}}_{H_u} \overline{\mathbf{5}}_{H_d} \mathbf{10}^i_M \tag{2.8}$$

from the Kähler potential. Again the coefficients may contain singlet VEVs.

Matter parity [19, 20] is a $\mathbb{Z}_2$ symmetry under which the matter fields are odd and the Higgs fields are even:

	$\overline{\mathbf{5}}^i_M, \mathbf{10}^i_M$	$\mathbf{5}_{H_u}, \overline{\mathbf{5}}_{H_d}$
P_M	-1	$+1$

(2.9)

P_M forbids all operators in Eqns. (2.7) and (2.8) except W^1 and W^3. The W^3 operator leads to neutrino masses via the Weinberg operator $LLHH$. This might still be allowed if it is generated at higher order in the singlets. W^1, on the other hand, is very tightly constrained and must be very strongly suppressed.

We will later see that when splitting the Higgs curves, some of the $\mathbf{10}$ multiplets will be split as well. So one could wonder whether it is possible that the W^1 operator is present at the $SU(5)$ level, but the split will be such that for the SM multiplets, there are no dangerous terms. To see that this is not the case, we write out W^1 in terms of SM representations,

$$\begin{aligned} W^1_{ijkl} \mathbf{10}^i_M \mathbf{10}^j_M \mathbf{10}^k_M \overline{\mathbf{5}}^l_M = {} & W^1_{ijkl} \overline{e}^i \overline{u}^j Q^k L^l + W^1_{ijkl} \overline{e}^i \overline{u}^j \overline{u}^k \overline{d}^l \\ & + W^1_{ijkl} \overline{u}^i Q^j Q^k \overline{d}^l + W^1_{ijkl} Q^i Q^j Q^k L^l \,. \end{aligned} \tag{2.10}$$

The first term, for example, is only absent for all permutations of i, j and k when the $\overline{\mathbf{5}}^l_M$ is split such that the chirality of L^l is zero. For the second term, however, the same argument applies for $\overline{d}^l$. Since this reasoning works for all l and because we must require that at least the sum of the chiralities over all l is minus three for both L and $\overline{d}$, we conclude that W^1 must be absent at the $SU(5)$ level.

3 Local Models with Matter Parity

This section outlines the steps that lead to the two local models presented in this work. To specify a concrete model one has to choose the monodromy, define the matter parity P_M, assign matter and Higgs fields to the curves and select a set of singlets that get a VEV. This list could give the impression that there is a lot of freedom in the attempt of building a local or semilocal model which incorporates matter parity. However, we can drastically reduce the number of options by imposing a few reasonable phenomenological requirements.

Our requirements are:

0. A matter parity $P_M \subset SU(5)_\perp$,
1. a heavy top quark, i.e. a tree-level rank-one up-type Yukawa coupling involving the third generation **10** curve,
2. absence of dimension-five proton decay via the W^1 operator,
3. masses for all quarks and leptons after switching on singlet VEVs.

We will start with general arguments that must be valid in the local as well as in the semilocal framework by showing in Section 3.1 that there are only two possible definitions of matter parity and essentially one choice of monodromy group. In the subsequent sections we will first consider case I, demonstrating in Section 3.2.1 that the way to assign matter fields to curves is very restricted when requiring the absence of proton decay. In Section 3.2.2 we discuss the possibilities to choose the down-type Higgs curve and we will see that this leads in fact to a unique local model for case I whose phenomenology will be examined in Section 3.2.3. Afterwards, in Section 3.3 a similar analysis will be performed for case II, where there is the possibility of enlarging the monodromy group, but without introducing qualitatively new features.

3.1 Matter Parity and Monodromy

We would first like to motivate the choice of the monodromy group and the definition of matter parity. We require that there is a tree-level coupling $\mathbf{10}_M\mathbf{10}_M\mathbf{5}_{H_u}$ that leads to a heavy top quark. Since the top quark and the anti-top quark both are in the same **10** representation, this can only be achieved provided the $\mathbf{10}_M$'s participating in the mass term are the same. Keeping in mind that the up-type Higgs has a charge of the form $-t_i - t_j$, the $\mathbf{10}_M$'s must have charges t_i and t_j for the mass term to be gauge invariant. Since $t_i \neq t_j$, see Eq. (2.3), this would imply that the **10**'s are different and thus to allow the top-anti-top quark coupling at least a $\mathbb{Z}_2$ monodromy is required. We choose its action to be $t_1 \leftrightarrow t_2$, so that the top quark generation is assigned to the corresponding curve $\mathbf{10}_1$ given in terms of the weight representation by $\{t_1, t_2\}$. Then there exists a tree-level up-type Yukawa coupling that leads to a heavy top quark provided we also fix the up-type Higgs curve $\mathbf{5}_{H_u}$ to be the curve with the charge $-t_1 - t_2$.

Our matter parity emerges from the $SU(5)_\perp$ and therefore must be defined in terms of the t_i. Each t_i can either contribute a factor of $+1$ or -1 and thus a formula for P_M can be written in full generality as $P_M = (-1)^{\alpha_1 t_1+\alpha_2 t_2+\alpha_3 t_3+\alpha_4 t_4+\alpha_5 t_5}$, where α_i takes the values 1 or 2. Other values for the α's would not give anything new because it is a $\mathbb{Z}_2$ symmetry. Note that the up-type Yukawa coupling will always be allowed by matter parity because the requirement of gauge invariance alone leads to $P_M(\text{up-type Yukawa coupling}) = (-1)^0$ since the t's cancel and this conclusion persists no matter how many singlets are inserted.

The requirement that the down-type Yukawa couplings $\mathbf{10}_M\overline{\mathbf{5}}_M\overline{\mathbf{5}}_{H_d}$ should be allowed does give a constraint on the matter parity definition though: Given that the $\mathbf{10}_M$ contributes a factor of t_i and the $\overline{\mathbf{5}}$'s contribute $t_j + t_k$ and $t_l + t_m$, all of which have a positive sign, this operator can, in contrast to the up-type Yukawa coupling, only be gauge invariant if all t's are different, so that we get $t_1+t_2+t_3+t_4+t_5$, which is zero according to Eq. (2.2). At the same time the desired down-type Yukawa coupling must have matter parity $+1$ to be permitted, which can only be achieved provided the number of t's with a prefactor of 2 in the matter parity definition is odd. Note that this fact remains true with any number of singlet insertions because the singlets all have charge assignments of the form $t_i - t_j$ and thus do not change the number of t's in the operator. When setting all five α's to 2 there will not be a single field left that we could identify with matter since matter must have $P_M = -1$. One option is to set only a single α to 2, which we choose to be α_5:

$$\text{Case I}: \quad P_M = (-1)^{t_1+t_2+t_3+t_4+2t_5}\,. \tag{3.1}$$

This model will be analysed in Section 3.2. Finally, having three prefactors of 2 in the matter parity definition forces us to build a model where three generations come from a single $\mathbf{10}_M$ curve, namely the top curve. We will examine the model corresponding to the matter parity

$$\text{Case II}: \quad P_M = (-1)^{t_1+t_2+2t_3+2t_4+2t_5} \tag{3.2}$$

in Section 3.3.

Now let us come back to the choice of the monodromy group. In case I, for P_M to be well-defined, t_5 must not be related to any other t. So there are only two options left which maintain the chance of building a model where the three generations of the standard model emerge from at least two curves. The first one is to enlarge the monodromy group such that another t lies in the same orbit as t_1 and t_2 and the second one is to additionally relate t_3 and t_4 by a $\mathbb{Z}_2$ monodromy. Both ideas are to be discarded because they are accompanied by the occurrence of the operator $W^1_{ijkl}\mathbf{10}^i_M\mathbf{10}^j_M\mathbf{10}^k_M\overline{\mathbf{5}}^l_M$, which leads to proton decay, see Section 2.2. The origin of this issue is that a gauge invariant W^1 operator also needs a sum over all five distinct t's, as it is the case for the down-type Yukawa couplings. More precisely, since the three $\mathbf{10}$'s in W^1 each add a t with a prefactor of one, for an allowed W^1 the $\overline{\mathbf{5}}$ must provide the remaining two t's, in particular a factor of t_5 that cannot get in via the $\mathbf{10}_M$'s. A $\overline{\mathbf{5}}$ always has a charge assignment $t_i + t_j$. So for it to have $P_M = -1$ and from our definition of matter parity it is evident that one of these t's must in fact be the t_5.

Now the only chance to avoid W^1 is to not identify one of the odd matter parity $\mathbf{10}$'s with SM matter. Since already with the $\mathbb{Z}_2$ monodromy there are only 3 odd parity curves left, it is evident that the monodromy group must not be enlarged any further.

In case II, t_3, t_4 and t_5 appear symmetrically in the definition of P_M. Here it is possible to mutually relate them by an arbitrary monodromy, but we will see later that this does not affect the phenomenology of the resulting model and therefore we will leave it at the monodromy relating t_1 and t_2.

3.2 Matter Parity Case I

3.2.1 Matter Curves and Singlet VEVs

Having fixed the monodromy group and a formula for matter parity,

$$P_M = (-1)^{t_1+t_2+t_3+t_4+2t_5} ,$$

we proceed with the discussion which fields to assign to the different curves. The aim of this selection is to prevent the appearance of baryon and lepton number violating operators. We have collected the curves, their charges and matter parities in Table 3.1. As already mentioned in the previous section, all dimension three, four and five baryon and lepton number violating operators apart from $W^3_{ij}\overline{\mathbf{5}}^i_M\overline{\mathbf{5}}^j_M\mathbf{5}_{H_u}\mathbf{5}_{H_u}$ and $W^1_{ijkl}\mathbf{10}^i_M\mathbf{10}^j_M\mathbf{10}^k_M\overline{\mathbf{5}}^l_M$ are forbidden by matter parity. Since the two up-type Higgs curves in W^3 contribute a factor $-2t_1-2t_2$ to the operator, W^3 is absent at tree level because these charges cannot be canceled by adding the t's for the other two curves. On the other hand, W^1 will appear at tree level, as we have argued above, unless we do not assign SM matter to some of the odd matter parity curves.

From Eq. (3.3), which lists all gauge invariant combinations involving the $\mathbf{10}$'s and $\mathbf{5}$'s with $P_M = -1$, one can see that it is possible to evade W^1 when not assigning SM fields to the curves $\mathbf{10}_2$ and $\mathbf{5}_5$ because if these two curves do not carry SM fields, there is no operator left that contains SM fields only[3].

$$\begin{aligned}&\mathbf{10}_1\mathbf{10}_1\mathbf{10}_2\overline{\mathbf{5}}_6\\&\mathbf{10}_1\mathbf{10}_1\mathbf{10}_3\overline{\mathbf{5}}_5\\&\mathbf{10}_1\mathbf{10}_2\mathbf{10}_3\overline{\mathbf{5}}_3\end{aligned} \tag{3.3}$$

Hence, the $\mathbf{10}$ and $\mathbf{5}$ curves carrying SM matter are fixed to be $\mathbf{10}_1$ and $\mathbf{10}_3$ as well as $\mathbf{5}_3$ and $\mathbf{5}_6$, respectively, because one can see from Table 3.1 that these are the only remaining fields with odd matter parity. Because of this we are forced to build a model where three fields emerge from two curves.

Addressing the even matter parity $\mathbf{5}$ curves, there are four possibilities to assign the down-type Higgs to one of the Higgs-like curves. The choice must be made such that after

[3]One could also pick $\mathbf{10}_3$ and $\mathbf{5}_6$ instead but this choice just amounts to a relabeling.

	Charge	Matter Parity	Assigned Fields
10 Curves			
$\mathbf{10}_1$	$\{t_1, t_2\}$	$-$	$\mathbf{10}_{\text{top}}$, possibly more
$\mathbf{10}_2$	t_3	$-$	no SM matter
$\mathbf{10}_3$	t_4	$-$	possible SM matter
$\mathbf{10}_4$	t_5	$+$	no SM matter
5 Curves			
$\mathbf{5}_{H_u}$	$-t_1 - t_2$	$+$	up-type Higgs
$\mathbf{5}_1$	$\{-t_1 - t_3, -t_2 - t_3\}$	$+$	Higgs-like
$\mathbf{5}_2$	$\{-t_1 - t_4, -t_2 - t_4\}$	$+$	Higgs-like
$\mathbf{5}_3$	$\{-t_1 - t_5, -t_2 - t_5\}$	$-$	possible SM matter
$\mathbf{5}_4$	$-t_3 - t_4$	$+$	Higgs-like
$\mathbf{5}_5$	$-t_3 - t_5$	$-$	no SM matter
$\mathbf{5}_6$	$-t_4 - t_5$	$-$	possible SM matter
Singlets			
$\mathbf{1}_1$	$\pm\{t_1 - t_3, t_2 - t_3\}$	$+$	no VEV
$\mathbf{1}_2$	$\pm\{t_1 - t_4, t_2 - t_4\}$	$+$	VEV possible
$\mathbf{1}_3$	$\pm\{t_1 - t_5, t_2 - t_5\}$	$-$	no VEV
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	no VEV
$\mathbf{1}_5$	$\pm(t_3 - t_5)$	$-$	no VEV
$\mathbf{1}_6$	$\pm(t_4 - t_5)$	$-$	no VEV
$\mathbf{1}_7$	$\{t_1 - t_2, t_2 - t_1\}$	$+$	VEV possible

Table 3.1: *List of curves, their charges and their matter parity values with the field assignment for case I.*

turning on VEVs for selected singlets the down quarks become massive without reintroducing operators that lead to proton decay. We do not give VEVs to odd matter parity singlets because this would break matter parity and reintroduce the successfully eliminated baryon and lepton number violating operators. W^1 will be generated immediately when VEVs are given to the singlets $\mathbf{1}_1$ or $\mathbf{1}_4$ because the matter $\mathbf{10}$'s have the charges t_1, t_2 and t_4 and the matter $\mathbf{5}$'s have the charges $\{t_1 + t_5, t_2 + t_5\}$ and $t_4 + t_5$. $\mathbf{1}_1$ and $\mathbf{1}_4$ have the charges $\pm\{t_1 - t_3, t_2 - t_3\}$ and $\pm(t_3 - t_4)$ and thus both contain a t_3 which is needed for W^1 to be gauge invariant. Therefore they must not get a VEV. Summing up, the aim is to select a down-type Higgs curve that gives masses to the down-type quarks using only VEVs for the singlets $\mathbf{1}_2$ and $\mathbf{1}_7$. The assignment of fields to the matter curves is summarised in the last column of Table 3.1.

3.2.2 Higgs Curves

In this section we will see that, when working in the purely local framework, requiring no proton decay and down-type masses at the same time singles out a unique model for case I. The main assumption is that the chiralities of the curves can be chosen at will while simultaneously the Higgs curves can be split correctly, that is, only the Higgs doublets remains light. In particular, we assume that all $\mathbf{10}$ and $\mathbf{5}$ curves apart from $\mathbf{10}_1$, $\mathbf{10}_3$, $\mathbf{5}_{H_u}$, $\overline{\mathbf{5}}_{H_d}$, $\overline{\mathbf{5}}_3$ and $\overline{\mathbf{5}}_6$ appear as vector-like pairs and can be given a high-scale mass. In Section 4 this point will be analysed in more detail.

The previous section tells us that, since the SM matter curves are fixed, the next important question is to which of the four possible even matter parity $\mathbf{5}$ curves $\mathbf{5}_{H_u}$, $\mathbf{5}_1$, $\mathbf{5}_2$ and $\mathbf{5}_4$ the down-type Higgs field is assigned. Table 3.2 lists all gauge invariant tree-level down-type Yukawa couplings for the different choices of the down-type Higgs curve.

Down-type Higgs curve	**Gauge invariant couplings**
$\overline{\mathbf{5}}_{H_u}$	—
$\overline{\mathbf{5}}_1$	$\overline{\mathbf{5}}_{H_d}\mathbf{10}_1\overline{\mathbf{5}}_6$ $\overline{\mathbf{5}}_{H_d}\mathbf{10}_3\overline{\mathbf{5}}_3$
$\overline{\mathbf{5}}_2$	—
$\overline{\mathbf{5}}_4$	$\overline{\mathbf{5}}_{H_d}\mathbf{10}_1\overline{\mathbf{5}}_3$

Table 3.2: *Gauge invariant down-type Yukawa terms for all possible choices of down-type Higgs curves for case I.*

Taking the down-type Higgs curve to be $\overline{\mathbf{5}}_1$ leads to a rank-two Yukawa matrix at tree level resulting in two heavy and one light generations, which is phenomenologically problematic. One can check that a particular split of the curves reduces the rank of the matrix to one or zero, but since with the spectral cover formalism it is not possible to realise the $\overline{\mathbf{5}}_1$ as the down-type Higgs curve anyway, as we will see later, we dismiss this option here and relegate more details to Appendix A .

Consider next the choice of $\overline{\mathbf{5}}_{H_u}$ or $\overline{\mathbf{5}}_2$ with charges $\{t_1+t_2\}$ and $\{t_1+t_4, t_2+t_4\}$, respectively. It is easy to see that in both cases there cannot be any down-type masses because the matter $\mathbf{10}$'s, $\mathbf{10}_1$ and $\mathbf{10}_3$, have charges $\{t_1, t_2\}$ and t_4, while the matter $\overline{\mathbf{5}}$'s, $\overline{\mathbf{5}}_3$ and $\overline{\mathbf{5}}_6$, have charges $\{t_1+t_5, t_2+t_5\}$ and t_4+t_5, and the only two singlets that we are allowed to give a VEV to, as discussed in the previous section, are $\mathbf{1}_2$ and $\mathbf{1}_7$ with charges $\pm\{t_1-t_4, t_2-t_4\}$ and $\{t_1-t_2, t_2-t_1\}$. For the down-type mass term to be gauge invariant, a sum over all five distinct t's is needed and with this choice it is obvious that the sum will

always lack a t_3 factor. Thus, the possibilities to select $\overline{\mathbf{5}}_{H_u}$ or $\overline{\mathbf{5}}_2$ as the down-type Higgs curve are excluded. Note that this conclusion holds even if the curves are eventually split.

This ultimately fixes the down-type Higgs curve to be $\overline{\mathbf{5}}_4$

$$\overline{\mathbf{5}}_{H_d} = \overline{\mathbf{5}}_4 \,, \tag{3.4}$$

which is favoured anyway because it leads to a rank-one down-type Yukawa matrix at tree level. Demanding that the quark which gets the large mass is the bottom quark then amounts to assigning the bottom quark generation to the curve $\overline{\mathbf{5}}_3$ and the light generations to the curve $\overline{\mathbf{5}}_6$,

$$\overline{\mathbf{5}}_{\text{bottom}} = \overline{\mathbf{5}}_3 \,, \quad \overline{\mathbf{5}}_{\text{down/strange}} = \overline{\mathbf{5}}_6 \,. \tag{3.5}$$

Recapitulating, the requirements of no proton decay, a heavy top quark and a down-type Yukawa matrix which has rank zero or one have guided us to a unique model. The next step is to explore its phenomenology.

3.2.3 Masses and Mixings

Let us now examine the Yukawa textures and Cabibbo–Kobayashi–Maskawa (CKM) matrix in the model that was just specified to see whether reasonable mass hierarchies and mixings in the quark sector can be achieved.

Choosing $\mathbf{10}_1$ to carry only the top generation and $\mathbf{10}_3$ the light generations, calculating the up-type Yukawa matrix including insertions of the singlets $\mathbf{1}_2$ and $\mathbf{1}_7$ leads, at leading order, to the following result.

$$Y^u_{ij} \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \,, \tag{3.6}$$

where

$$\epsilon = \frac{\langle X_2 \rangle}{M_*} \,. \tag{3.7}$$

Here $\langle X_2 \rangle$ is the VEV for the field assigned to the curve $\mathbf{1}_2$, which is suppressed by the winding scale M_* which is also the GUT scale for local models [28]. The VEV for $\mathbf{1}_7$ will only appear at higher order and so can be ignored at this level.

It is important to note that there are order-one prefactors in front of each entry, which depend on the geometry of the different curves and also come from integrating out heavy states in the case of elements with singlet insertions. Within this framework these prefactors cannot be determined, but at this point we are not interested in the details of the matrices. Instead, we would like to see if they show acceptable patterns.

For the down-type Yukawa matrix we get a similar result:

$$Y_{ij}^d \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} . \tag{3.8}$$

Since these matrices are both approximately diagonal, we can use the simplified formulae for the mixing angles in the CKM matrix [29]

$$s_{ij}^{\mathrm{CKM}} \simeq s_{ij}^d - s_{ij}^u \,, \tag{3.9}$$

where

$$s_{12}^{u/d} = \frac{Y_{12}^{u/d}}{Y_{22}^{u/d}} \,, \quad s_{13}^{u/d} = \frac{Y_{13}^{u/d}}{Y_{33}^{u/d}} \,, \quad s_{23}^{u/d} = \frac{Y_{23}^{u/d}}{Y_{33}^{u/d}} \,, \tag{3.10}$$

and we arrive at

$$V_{\mathrm{CKM}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} . \tag{3.11}$$

These results show that in our model where the Yukawa matrices and the CKM matrix are described by a single parameter $\langle X_2 \rangle$, a mass can be given to all generations and in addition there is also some mixing. Neither the Yukawa matrices nor the CKM matrix match the SM data very well, but this was not expected because in our setup three generations come from only two curves and thus there will always be a certain degeneracy in the entries of the Yukawa matrices and thus also in the CKM matrix.

3.3 Matter Parity Case II

In this section we analyse the matter parity case II along the lines of the previous section: We will first clarify the possible field assignment and then discuss in Section 3.3.2 the down-type Higgs sector and the resulting Yukawa textures.

The matter parity in case II is defined as

$$P_M = (-1)^{t_1+t_2+2t_3+2t_4+2t_5} \,.$$

It will turn out that one has in general more freedom in this model because the $SU(5)_\perp$ charges split into two more or less decoupled groups, $t_{\mathrm{even}} = \{t_3, t_4, t_5\}$ and $t_{\mathrm{odd}} = \{t_1, t_2\}$, such that the possible Higgs $\mathbf{5}$ curves involve two t_{even}'s and the matter $\mathbf{5}$ curves involve one t_{even} and one t_{odd}. Furthermore, the positive matter parity singlets do not mix t_{even} and t_{odd}.

As in case I, we find a basically unique model. There is only one matter $\mathbf{10}$ curve, and both Higgses are unique (up to a relabeling of the t_{even}). The only freedom is in the choice of matter $\overline{\mathbf{5}}$ curves: One can choose one, two or three curves for the three generations, or alternatively identify some of these by an extended monodromy. However, this will not give qualitatively new features.

	Charge	Matter Parity	Assigned Fields
10 Curves			
$\mathbf{10}_1$	$\{t_1, t_2\}$	$-$	all families
$\mathbf{10}_2$	t_3	$+$	no SM matter
$\mathbf{10}_3$	t_4	$+$	no SM matter
$\mathbf{10}_4$	t_5	$+$	no SM matter
5 Curves			
$\mathbf{5}_{H_u}$	$-t_1 - t_2$	$+$	up-type Higgs
$\mathbf{5}_1$	$\{-t_1 - t_3, -t_2 - t_3\}$	$-$	possible SM matter
$\mathbf{5}_2$	$\{-t_1 - t_4, -t_2 - t_4\}$	$-$	possible SM matter
$\mathbf{5}_3$	$\{-t_1 - t_5, -t_2 - t_5\}$	$-$	possible SM matter
$\mathbf{5}_4$	$-t_3 - t_4$	$+$	Higgs-like
$\mathbf{5}_5$	$-t_3 - t_5$	$+$	Higgs-like
$\mathbf{5}_6$	$-t_4 - t_5$	$+$	Higgs-like
Singlets			
$\mathbf{1}_1$	$\pm\{t_1 - t_3, t_2 - t_3\}$	$-$	no VEV
$\mathbf{1}_2$	$\pm\{t_1 - t_4, t_2 - t_4\}$	$-$	no VEV
$\mathbf{1}_3$	$\pm\{t_1 - t_5, t_2 - t_5\}$	$-$	no VEV
$\mathbf{1}_4$	$\pm(t_3 - t_4)$	$+$	VEV possible
$\mathbf{1}_5$	$\pm(t_3 - t_5)$	$+$	VEV possible
$\mathbf{1}_6$	$\pm(t_4 - t_5)$	$+$	VEV possible
$\mathbf{1}_7$	$\{t_1 - t_2, t_2 - t_1\}$	$+$	VEV possible

Table 3.3: *List of curves, their charges and their matter parity values with the field assignment for case II.*

3.3.1 Matter Assignment

In the case at hand the different curves have even and odd matter parity as displayed in Table 3.3. Since there is only one odd matter parity $\mathbf{10}$ curve in case II, all three generations of SM fields that belong to the $\mathbf{10}$ representation have to be assigned to $\mathbf{10}_1$. Note, however, that one can choose the matter in the $\overline{\mathbf{5}}$ representation to come from one, two or three $\overline{\mathbf{5}}$ curves.

The situation with the W^1 operator has improved compared to case I: Using t_{even} and t_{odd} defined above, the operator $\mathbf{10}_M\mathbf{10}_M\mathbf{10}_M\overline{\mathbf{5}}_M$ then has the charge $4t_{\text{odd}} + t_{\text{even}}$. Therefore, with this choice of matter parity and under the assumption that VEVs are given only to matter parity even singlets, W^1 cannot be generated no matter which and how many singlets are inserted. This statement is also completely independent of the assignment of fields to the curves, which is only constrained by the matter parity and shown in Table 3.3.

3.3.2 Higgs Assignment and Flavour

Since there is only one $\mathbf{10}$ matter curve, there is only one up-type Yukawa coupling, $\mathbf{5}_{H_u}\mathbf{10}_1\mathbf{10}_1$, which at tree level leads to a Yukawa matrix

$$Y^u_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.12}$$

We can get singlet contributions from e.g. the VEV of $\mathbf{1}_7$, or various higher powers of other singlet VEVs. This gives a generic form of the Yukawa matrix as

$$Y^u_{ij} \sim \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} . \tag{3.13}$$

Here the ϵ's again involve order-one coefficients.

Turning to the down-type Yukawa couplings, there are four Higgs-like $\overline{\mathbf{5}}$ curves. Having the down-type Higgs on the same curve as the up-type Higgs, there will be no generation of down-type masses at any level as can be seen from the charges: The reason for the absence is the same as for the W^1 operator. We get for the coupling $\mathbf{10}_M\overline{\mathbf{5}}_M\overline{\mathbf{5}}_{H_u}$ again $4t_{\text{odd}} + t_{\text{even}}$ which, as already stated, can never be made gauge invariant by singlet insertions. All other choices for the down-type Higgs curve are equivalent, given that this model is invariant under permutations of t_3, t_4 and t_5 and the three remaining Higgs-like curves and possible $\overline{\mathbf{5}}$ matter curves only involve these charges. So let us choose $\overline{\mathbf{5}}_{H_d} = \overline{\mathbf{5}}_4$. In order to have a tree-level coupling of the $\mathbf{10}_1$ curve to any down-type quark, we have to assign matter to the curve $\mathbf{5}_3$. It is also possible to start with a down-type Yukawa matrix of rank zero at tree level and generate all masses through singlet insertions. The allowed couplings to lowest order in the singlets between the down-type Higgs curve and the candidate SM matter $\overline{\mathbf{5}}$ curves are:

$$\mathbf{10}_1\overline{\mathbf{5}}_3\overline{\mathbf{5}}_{H_d}, \quad \mathbf{10}_1\overline{\mathbf{5}}_1\overline{\mathbf{5}}_{H_d}\mathbf{1}_5, \quad \mathbf{10}_1\overline{\mathbf{5}}_1\overline{\mathbf{5}}_{H_d}\mathbf{1}_4\mathbf{1}_6, \quad \mathbf{10}_1\overline{\mathbf{5}}_2\overline{\mathbf{5}}_{H_d}\mathbf{1}_6, \quad \mathbf{10}_1\overline{\mathbf{5}}_2\overline{\mathbf{5}}_{H_d}\mathbf{1}_4\mathbf{1}_5 \tag{3.14}$$

Depending on which singlets get a VEV and how the SM generations are assigned to the three $\overline{\mathbf{5}}$ matter curves, one can arrive at different down-type Yukawa matrices. Starting with a rank-one matrix at tree level, one can assign the bottom quark generation to $\mathbf{5}_3$, and the first and second generation to $\mathbf{5}_1$ and $\mathbf{5}_2$, respectively. Switching on VEVs for $\mathbf{1}_5$ and $\mathbf{1}_4$, one arrives at a down-type Yukawa matrix, where some entries are generated with only one singlet insertion and others with two:

$$Y^d_{ij} \sim \begin{pmatrix} \epsilon_5 & \epsilon_5 & \epsilon_5 \\ \epsilon_5\epsilon_4 & \epsilon_5\epsilon_4 & \epsilon_5\epsilon_4 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.15}$$

Switching on VEVs for $\mathbf{1}_5$ and $\mathbf{1}_6$, all entries involving $\overline{d}$ and $\overline{b}$ are generated with only one singlet insertion:

$$Y_{ij}^d \sim \begin{pmatrix} \epsilon_5 & \epsilon_5 & \epsilon_5 \\ \epsilon_6 & \epsilon_6 & \epsilon_6 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.16}$$

Of course, there exists also the option to assign both light generations to one curve. Then, one singlet insertion is sufficient to get all couplings.

If one starts with a rank-zero matrix at tree level, all matter must be assigned to $\mathbf{5}_1$ and $\mathbf{5}_2$. Choosing $\mathbf{5}_1$ to carry the bottom generation and $\mathbf{5}_2$ the other generations, one can for example realise the matrix

$$Y_{ij}^d \sim \begin{pmatrix} \epsilon_5\epsilon_4 & \epsilon_5\epsilon_4 & \epsilon_5\epsilon_4 \\ \epsilon_5\epsilon_4 & \epsilon_5\epsilon_4 & \epsilon_5\epsilon_4 \\ \epsilon_5 & \epsilon_5 & \epsilon_5 \end{pmatrix} \tag{3.17}$$

or the same matrix with $\epsilon_5\epsilon_4$ replaced by ϵ_6.

Since they cannot be determined within this framework and we are not aiming at presenting a detailed discussion of flavour, we will now move on to the question whether the models presented in this section can be realised in a more global framework.

4 Semilocal Realisation

In this section we attempt to extend our local models to semilocal models, which would be the first step in searching for a global realisation. The important improvement of the semilocal framework, where the whole GUT surface S_{GUT} is considered, is that the chiral spectrum of a model can be calculated explicitly. The chiralities of the curves are determined by the restrictions of two kinds of fluxes. One of them is turned on along the $U(1)$'s that remain after imposing the action of the monodromy group (i.e. along the transverse branes) and can only influence the chiralities of full GUT multiplets. The other flux is the hypercharge flux, which can split the multiplets. The latter, as opposed to the $U(1)$ flux, is confined to S_{GUT} and thus, as we will see, leads to strong constraints on the spectrum of semilocal models. The aim of Section 4.1 is, essentially following [15], to find a formula for the chiralities of the various fields in terms of the restrictions of the above fluxes. Therefore one needs to know the homology classes of the curves, which in turn can be calculated using the spectral cover approach. In Section 4.2 we will perform explicit calculations using the spectral cover results and show that our local models, presented in Section 3, unfortunately have no semilocal realisation. One should, however, keep in mind that, as was noted in [25], the spectral cover approach used here is not necessarily the most general framework. Thus, to exclude the local models once and for all, further studies seem to be necessary.

4.1 Summary of Spectral Cover Results

In Section 2 we discussed how the information about the monodromy group is contained in the deformation parameters b_k appearing in Eq. (2.4). Another elegant way to handle the monodromy data is to work with the spectral cover. It is defined to be the surface given by the constraint

$$C_{10} = b_0 U^5 + b_2 V^2 U^3 + b_3 V^3 U^2 + b_4 V^4 U + b_5 V^5 = 0\,, \tag{4.1}$$

where U and V are homogeneous coordinates of the projective threefold

$$X = \mathbb{P}(\mathcal{O}_{S_{GUT}} \oplus K_{S_{GUT}})\,. \tag{4.2}$$

$\mathcal{O}_{S_{GUT}}$ and $K_{S_{GUT}}$ denote the trivial and the canonical bundle on S_{GUT}, respectively. The monodromy is encoded in the factorisation of C_{10} because the number of $U(1)$'s that remain independent is in general one less than the number of factors of C_{10}. To visualise how this comes about one can picture all **10** curves to be one single **10** curve on that five-sheeted spectral cover. The different sheets of that cover get connected by branch cuts so that it breaks into slices, each of which is associated to a factor of C_{10} and corresponds to one **10** curve. Concretely, one can locally define a parameter $s = U/V$ and the five roots of Eq. (4.1), written as a polynomial in s, will then correspond to the five t_i. It is now clear that in order to realise the $\mathbb{Z}_2$ monodromy, we need the following factorisation into four parts, where the curves $\mathbf{10}_1$ and $\mathbf{10}_2$ lift to a single curve on the spectral cover:

$$C_{10} = \left(a_1 V^2 + a_2 VU + a_3 U^2\right)(a_4 V + a_7 U)(a_5 V + a_8 U)(a_6 V + a_9 U) = 0 \tag{4.3}$$

It is possible to solve for the a_i in terms of the b_i and to calculate their homology classes. Note that Eq. (2.4) implies that the b_k are sections of line bundles with first Chern class $\eta - kc_1$, where

$$\eta = 6c_1 - t\,, \quad t = -c_1(N)\,, \quad c_1 = c_1(S_{GUT})\,, \tag{4.4}$$

with N the normal bundle of S_{GUT}. Since there are six b's, but nine a's, three bundles remain unspecified and are denoted χ_7, χ_8 and χ_9 corresponding to a_7, a_8 and a_9.

The constraint one gets from

$$b_1 = t_1 + t_2 + t_3 + t_4 + t_5 = 0 \tag{4.5}$$

implies

$$a_2 a_7 a_8 a_9 + a_3 a_6 a_7 a_8 + a_3 a_5 a_7 a_9 + a_3 a_4 a_8 a_9 = 0 \tag{4.6}$$

and is nontrivial. It can be solved by the ansatz

$$\begin{aligned} a_2 &= -c\,(a_6 a_7 a_8 + a_5 a_7 a_9 + a_4 a_8 a_9) \\ a_3 &= c\; a_7 a_8 a_9\,, \end{aligned} \tag{4.7}$$

Section	c_1(Bundle)
a_1	$\eta - 2c_1 - \tilde{\chi}$
a_2	$\eta - c_1 - \tilde{\chi}$
a_3	$\eta - \tilde{\chi}$
a_4	$-c_1 + \chi_7$
a_5	$-c_1 + \chi_8$
a_6	$-c_1 + \chi_9$
a_7	χ_7
a_8	χ_8
a_9	χ_9

Table 4.1: *The first Chern classes of the line bundles corresponding to the* a_i.

without inducing non-Kodaira singularities, as was shown in [30]. This, however, does not need to be the only solution and might thus not constitute the most general one. Here, the homology class $[c]$ is introduced which is given by

$$[c] = \eta - 2\tilde{\chi}\,, \tag{4.8}$$

where $\tilde{\chi} = \chi_7 + \chi_8 + \chi_9$. Table 4.1 summarises the Chern classes of the various bundles for all a_i.

Next, we need to determine the matter curves in terms of the a_i. Eq. (2.3) tells us that the 10 curves are given by $t_i = 0$ and this implies $b_5 = t_1t_2t_3t_4t_5 = 0$, which is in turn the coefficient of V^5 and must also be equal to $a_1a_4a_5a_6$, as one can see from Eq. (4.3). Therefore, one concludes that the **10** curves are given by $a_k = 0$, where $k = 1, 4, 5, 6$.

In order to determine the equations for the **5** curves, we need to plug (4.7) into the defining polynomial for the **5** curves (2.5). This gives

$$\begin{aligned} P_5 = {} & (a_5a_7 + a_4a_8)\,(a_6a_7 + a_4a_9)\,(a_6a_8 + a_5a_9) \\ & \times (a_6a_7a_8 + a_5a_7a_9 + a_4a_8a_9) \\ & \times (a_1 - ca_5a_6a_7 - ca_4a_6a_8) \\ & \times (a_1 - ca_5a_6a_7 - ca_4a_5a_9) \\ & \times (a_1 - ca_4a_6a_8 - ca_4a_5a_9)\ , \end{aligned} \tag{4.9}$$

and we arrive at Table 4.2, which displays the curves, their $SU(5)_\perp$ charges and their defining equations in terms of the a_i as well as the resulting homology classes.

Now we can finally specify the spectrum in terms of the restrictions of the $U(1)$ fluxes and the hypercharge flux to the curves. If one denotes by the integers M and N_Y the restriction of the $U(1)$ fluxes and the hypercharge flux to a curve, then the **5** curves get split in the following way:

$$\begin{aligned} n_{(3,1)_{-1/3}} - n_{(\bar{3},1)_{+1/3}} &= M_{\mathbf{5}}\,, \\ n_{(1,2)_{+1/2}} - n_{(1,2)_{-1/2}} &= M_{\mathbf{5}} + N_Y\,, \end{aligned} \tag{4.10}$$

Curve	$SU(5)_\perp -$ Charge	Equation	Homology
$\mathbf{5}_{H_u}$	$-2t_1$	$a_6a_7a_8+a_5a_7a_9+a_4a_8a_9$	$-c_1+\tilde{\chi}$
$\mathbf{5}_1$	$-t_1-t_3$	$a_1-ca_4a_6a_8-ca_4a_5a_9$	$\eta-2c_1-\tilde{\chi}$
$\mathbf{5}_2$	$-t_1-t_4$	$a_1-ca_5a_6a_7-ca_4a_5a_9$	$\eta-2c_1-\tilde{\chi}$
$\mathbf{5}_3$	$-t_1-t_5$	$a_1-ca_5a_6a_7-ca_4a_6a_8$	$\eta-2c_1-\tilde{\chi}$
$\mathbf{5}_4$	$-t_3-t_4$	$a_5a_7+a_4a_8$	$-c_1+\chi_7+\chi_8$
$\mathbf{5}_5$	$-t_3-t_5$	$a_6a_7+a_4a_9$	$-c_1+\chi_7+\chi_9$
$\mathbf{5}_6$	$-t_4-t_5$	$a_6a_8+a_5a_9$	$-c_1+\chi_8+\chi_9$
$\mathbf{10}_1$	t_1	a_1	$\eta-2c_1-\tilde{\chi}$
$\mathbf{10}_2$	t_3	a_4	$-c_1+\chi_7$
$\mathbf{10}_3$	t_4	a_5	$-c_1+\chi_8$
$\mathbf{10}_4$	t_5	a_6	$-c_1+\chi_9$

Table 4.2: *Matter curves and their homology classes.*

and for the **10** curves we have

$$\begin{aligned} n_{(3,2)_{+1/6}} - n_{(\bar{3},2)_{-1/6}} &= M_{\mathbf{10}}\,, \\ n_{(\bar{3},1)_{-2/3}} - n_{(3,1)_{+2/3}} &= M_{\mathbf{10}} - N_Y\,, \\ n_{(1,1)_{+1}} - n_{(1,1)_{-1}} &= M_{\mathbf{10}} + N_Y\,. \end{aligned} \tag{4.11}$$

As was already mentioned in the introduction to this section, the $U(1)$ fluxes are turned on along the branes which intersect S_{GUT} and thus cannot be determined even in the semilocal approach. We are therefore allowed to treat the M's as free parameters up to two constraints:

$$\sum M_{\mathbf{10}} + \sum M_{\mathbf{5}} = 0\,, \qquad\qquad M_{\mathbf{10}_1} = -(M_{\mathbf{5}_1}+M_{\mathbf{5}_2}+M_{\mathbf{5}_3})\,. \tag{4.12}$$

The first equation follows from the tracelessness condition for the four $U(1)$ fluxes $\sum_i F_{U(1)_i} = 0$ and implies anomaly cancellation (see also [31]). The second one holds because if one defines the flux restrictions of the three remaining independent $U(1)$ fluxes to the **5** curves $\mathbf{5}_1$, $\mathbf{5}_2$ and $\mathbf{5}_3$ to be $M_{-t_1-t_3} = M_{\mathbf{5}_1}$, $M_{-t_1-t_4} = M_{\mathbf{5}_2}$ and $M_{-t_1-t_5} = M_{\mathbf{5}_3}$, we can express the $U(1)$ flux restrictions to $\mathbf{10}_1$ as $M_{\mathbf{10}_1} = M_{t_1} = M_{-(-3t_1+2t_1)} = M_{-(-3t_1-t_3-t_4-t_5)} = M_{-(-t_1-t_3-t_1-t_4-t_1-t_5)} = -(M_{\mathbf{5}_1}+M_{\mathbf{5}_2}+M_{\mathbf{5}_3})$.

For the hypercharge flux, as anticipated earlier, there are more constraints because one must prevent it from receiving a Green–Schwarz mass, which is only possible if the flux is switched on exclusively along 2-cycles in S_{GUT} which are homologically trivial as two-cycles in the CY fourfold. This requirement leads to the constraints

$$F_Y \cdot c_1 = 0\,, \qquad\qquad F_Y \cdot \eta = 0\,. \tag{4.13}$$

It is interesting to note that from this it follows that

$$\sum_{\mathbf{5}} N = \sum_{\mathbf{10}} N = 0\,, \tag{4.14}$$

	N_Y	M
10 Curves		
$\mathbf{10}_1$	$-\widetilde{N}$	$-(M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_3})$
$\mathbf{10}_2$	N_7	$M_{\mathbf{10}_2}$
$\mathbf{10}_3$	N_8	$M_{\mathbf{10}_3}$
$\mathbf{10}_4$	N_9	$M_{\mathbf{10}_4}$
5 Curves		
$\mathbf{5}_{H_u}$	$\widetilde{N}$	$M_{\mathbf{5}_{H_u}}$
$\mathbf{5}_1$	$-\widetilde{N}$	$M_{\mathbf{5}_1}$
$\mathbf{5}_2$	$-\widetilde{N}$	$M_{\mathbf{5}_2}$
$\mathbf{5}_3$	$-\widetilde{N}$	$M_{\mathbf{5}_3}$
$\mathbf{5}_4$	$N_7 + N_8$	$M_{\mathbf{5}_4}$
$\mathbf{5}_5$	$N_7 + N_9$	$M_{\mathbf{5}_5}$
$\mathbf{5}_6$	$N_8 + N_9$	$M_{\mathbf{5}_6}$

Table 4.3: *Restrictions of hypercharge and $U(1)$ fluxes to the curves.*

so the fields in the representations $n_{(1,2)_{+1/2}}$, $n_{(\bar{3},1)_{-2/3}}$ and $n_{(1,1)_{+1}}$ have no net chirality.

Regarding the column of Table 4.2 that displays the homology classes, we see that the hypercharge restrictions to the curves are determined solely by N_7, N_8 and N_9. The final values for N_Y and M, needed for the calculation of concrete spectra that will be performed in the next section, is shown in Table 4.3, where $\widetilde{N} = N_7 + N_8 + N_9$.

It is important to note that a split of the up-type Higgs curve inevitably leads to a split of the $\mathbf{10}_1$ curve.

4.2 Semilocal Embedding of Case I

Using the above spectral cover results and the setup established so far, we will now start to calculate the concrete spectrum. The **10** curves that accommodate SM matter are fixed to be $\mathbf{10}_1$ and $\mathbf{10}_3$. We require that there are three net **10**'s after splitting the curves. This leads to the requirements that

$$M_{\mathbf{10}_1} + M_{\mathbf{10}_3} = 3\,, \qquad\qquad N_7 + N_9 = 0\,. \tag{4.15}$$

Furthermore, we require $M_{\mathbf{10}_1} \geq 1$ and $M_{\mathbf{10}_1} + N_8 \geq 0$ to have a heavy top quark. Similarly, we find for the $\overline{\mathbf{5}}$ curves that

$$M_{\mathbf{5}_3} + M_{\mathbf{5}_6} = -3\,, \qquad\qquad N_7 = 0\,. \tag{4.16}$$

Hence, also $N_9 = 0$, and the only remaining parameter that can be used to split some curves is N_8.

	$n_{(3,1)_{-1/3}} - n_{(\bar{3},1)_{+1/3}}$	$n_{(1,2)_{+1/2}} - n_{(1,2)_{-1/2}}$
$\mathbf{5}_{H_u}$	$M_{\mathbf{5}_{H_u}}$	$M_{\mathbf{5}_{H_u}} + N_8$
$\mathbf{5}_1$	$M_{\mathbf{5}_1}$	$M_{\mathbf{5}_1} - N_8$
$\mathbf{5}_2$	$M_{\mathbf{5}_2}$	$M_{\mathbf{5}_2} - N_8$
$\mathbf{5}_4$	$M_{\mathbf{5}_4}$	$M_{\mathbf{5}_4} + N_8$

Table 4.4: *Chiralities in terms of $U(1)$ and hypercharge flux restrictions for the Higgs-like* **5** *curves after imposing the matter sector constraints for case I.*

Let us continue with the other two **10** curves that are not associated with SM matter and should therefore better have no net chirality. Using Table 4.3, one sees that this can be achieved easily by simply setting

$$M_{\mathbf{10}_2} = M_{\mathbf{10}_4} = 0\,. \tag{4.17}$$

Since we do not want zero modes from the $\mathbf{5}_5$ curve, we also set $M_{\mathbf{5}_5} = 0$. Thus, we can achieve a satisfactory matter sector.

We now turn to the Higgses. The chiralities of the Higgs-like **5** curves are shown in Table 4.4 in terms of the $U(1)$ and hypercharge flux restrictions. The M's are constrained by the condition (4.12) which now reads

$$M_{\mathbf{5}_{H_u}} + M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_4} = 0\,. \tag{4.18}$$

The split of the Higgs-like **5** curves is controlled by the parameter N_8, as was already noted above. We need one down-type Higgs doublet and one up-type Higgs doublet. The easiest way to realise this is if the down-type Higgs doublet, being located on a $\overline{\mathbf{5}}$, has a chirality of -1 whereas the up-type Higgs doublet has a chirality of $+1$. Neither the down-type Higgs nor the up-type Higgs must have a light triplet. Since we always assume that fields which appear in vector-like pairs become massive, the simplest way to get rid of the triplets would be to set the corresponding M's to zero.

However, we can possibly tolerate triplets if they become heavy with VEVs switched on. Giving a VEV to the singlet $\mathbf{1}_2$, there are two allowed couplings between the four **5** curves of interest:

$$\overline{\mathbf{5}}_1\mathbf{5}_4\mathbf{1}_2\,, \tag{4.19}$$

$$\mathbf{5}_{H_u}\overline{\mathbf{5}}_2\mathbf{1}_2\,. \tag{4.20}$$

Hence we can pairwise decouple a triplet from the $\mathbf{5}_{H_u}$ with an antitriplet from the $\mathbf{5}_2$ curve. At the same time, we do not want to decouple the up-type Higgs doublet. Formulating these requirements in terms of the flux restrictions, we have for the up-type Higgs

$$\begin{aligned} \text{doublets:} \quad & M_{\mathbf{5}_{H_u}} + N_8 + (M_{\mathbf{5}_2} - N_8) = 1\,, \\ \text{triplets:} \quad & M_{\mathbf{5}_{H_u}} + M_{\mathbf{5}_2} = 0\,, \end{aligned} \tag{4.21}$$

(a) Parameter Choice (4.22)

	Triplets	Doublets
$\mathbf{5}_5$	0	0
$\mathbf{5}_{H_u}$	0	1
$\mathbf{5}_1$	0	-1
$\mathbf{5}_2$	0	-1
$\mathbf{5}_4$	0	1

(b) Parameter Choice (4.24)

	Triplets	Doublets
$\mathbf{5}_5$	0	0
$\mathbf{5}_{H_u}$	0	1
$\mathbf{5}_1$	1	0
$\mathbf{5}_2$	1	0
$\mathbf{5}_4$	-2	-1

Table 4.5: *Two possible splits of the Higgs curves for case I.*

which is a contradiction. Therefore, one generically gets a spectrum which contains additional exotic fields [14], or no up-type Higgs. Note that, since both the $\mathbf{5}_{H_u}$ and the $\mathbf{5}_2$ are split with $\widetilde{N}$, this result is independent of the hypercharge flux.

A loophole would be to choose $\overline{\mathbf{5}}_2$ as the down-type Higgs curve. Then the spectrum is free of exotics, but the coupling (4.20) is nothing but a μ-term. Furthermore, as discussed before, the $\overline{\mathbf{5}}_2$ curve has no t_3 factor, so there are no down-type masses in this model. One can nevertheless realise this spectrum with the parameter choice

$$M_{\mathbf{5}_{H_u}} = M_{\mathbf{5}_1} = M_{\mathbf{5}_2} = M_{\mathbf{5}_4} = 0\,, \quad N_8 = 1\,. \tag{4.22}$$

The remaining doublets from $\mathbf{5}_1$ and $\mathbf{5}_4$ can be decoupled by the term (4.19).

If we want to embed the model from Section 3.2.2, which has $\mathbf{5}_4$ as the down-type Higgs, we can use a similar argument as in Eq. (4.21) to arrive at the mutually contradicting constraints

$$\begin{aligned} \text{doublets:}\quad & M_{\mathbf{5}_4} + N_8 + (M_{\mathbf{5}_1} - N_8) = -1\,, \\ \text{triplets:}\quad & M_{\mathbf{5}_4} + M_{\mathbf{5}_1} = 0\,. \end{aligned} \tag{4.23}$$

An example spectrum one can get using $\overline{\mathbf{5}}_4$ as the down-type Higgs curve is shown in Table 4(b) with parameters

$$M_{\mathbf{5}_{H_u}} = 0, \quad M_{\mathbf{5}_1} = M_{\mathbf{5}_2} = 1\,, \quad M_{\mathbf{5}_4} = -2, \quad N_8 = 1\,. \tag{4.24}$$

$\mathbf{5}_4$ has the desired doublet but also two triplets, one of which can be combined with the triplet of the $\mathbf{5}_1$ via the coupling $\overline{\mathbf{5}}_1\mathbf{5}_4\mathbf{1}_2$. The other one, however, will remain light and apart from that there is another unwanted triplet in the $\mathbf{5}_2$.

4.3 Semilocal Embedding of Case II

Since there is only one $\mathbf{10}$ curve in this model that can carry SM matter, we have to require

$$M_{\mathbf{10}_1} = 3, \quad N_{\mathbf{10}_1} = -\widetilde{N} = 0. \tag{4.25}$$

The first condition is unproblematic because it implies

$$M_{\mathbf{5}_1} + M_{\mathbf{5}_2} + M_{\mathbf{5}_3} = -3, \tag{4.26}$$

which is exactly what we need since $\overline{\mathbf{5}}_1$, $\overline{\mathbf{5}}_2$ and $\overline{\mathbf{5}}_3$ are the possible matter curves. Furthermore, $\widetilde{N} = 0$ implies that the matter curves are not split, so we end up with a reasonable matter sector. On the other hand, $\widetilde{N} = 0$ inhibits a split of the up-type Higgs because $N_{H_u} = \widetilde{N}$, see Table 4.3. There is also no way of coupling the up-type Higgs curve to some other even matter parity Higgs curve to make the triplet heavy because the coupling has the charges $-2t_{\text{even}} + 2t_{\text{even}}$ in terms of the notation introduced in Section 3.3.1, which cannot be cancelled with even matter parity singlets. Therefore, we can conclude that also in case II it is not possible to arrive at a satisfying spectrum while giving masses to quarks and leptons.

Note that because of the way the hypercharge flux restricts to the Higgs-like curves, it is not possible to achieve a satisfactory Higgs sector. This is true in both matter parity cases, and even when allowing for exotics from the matter sector. Hence, it is ultimately the problem of doublet-triplet splitting which prohibits a semilocal embedding of the local models.

5 Conclusions and Outlook

The incorporation of GUTs in string theory aims at a consistent ultraviolet completion of all fundamental interactions including gravity. Such a consistency can only be assured if we have a consistent global string theory construction. One might still ask the question whether there are some properties of particle physics that could be studied in a bottom-up approach at a local level. In F-theory such a local description concerns a d=4 spacetime (point in extra dimensions) or a semilocal description (d=8 spacetime with four extra dimensions). Local descriptions give more freedom for model building but might not have valid ultraviolet completions and could thus be inconsistent.

In the present paper we have analysed the local E_8 point for the construction of an $SU(5)$ GUT and identified exactly two models that are consistent with sufficient proton stability and nontrivial masses for all quarks and leptons. They might be candidates for a realistic string version of the MSSM, although some aspects (such as Majorana neutrino masses) have not yet been addressed. It is interesting to see that proton stability can be implemented at the local point. This is in contrast to results in the heterotic theory where such mechanisms required some amount of nonlocality within the known consistent global constructions [32, 33].

Unfortunately the two local models mentioned above do not possess a consistent global completion. Our analysis using spectral cover techniques proves the inconsistency of the otherwise acceptable local models even at the level of the semilocal construction. This is one of the central results of our analysis. Other studies of the E_8 point [15–17, 27, 34] have

assumed that the existence of P_M requires a nonlocal mechanism. This implies that a crucial aspect of the MSSM is not provided by the local point and thus undermines its predictive power.

Recently, it has been argued [25] that the spectral cover considerations might possibly not be the only way to include semilocal effects[4]. It might be interesting to see whether more general approaches could validate the two models we have identified (maybe even without the need for nonlocalities). More work in this direction is surely needed. Still it seems that there is no alternative to global consistent model building. We cannot trust the predictions of local models as long as they are not confirmed by global constructions. In a more positive interpretation this tells us that string theory is more than just "bottom-up" model building and that we can learn nontrivial things for particle physics from the full string theory.

Acknowledgements

We are grateful to Thomas Grimm for helpful discussions. This work was partially supported by the SFB-Transregio TR33 "The Dark Universe" (Deutsche Forschungsgemeinschaft) and the European Union 7th network program "Unification in the LHC era" (PITN-GA-2009-237920).

[4]Note added: While finishing this work, the paper [35] appeared which discusses Yukawa couplings in the T-brane setup.

A $\overline{\mathbf{5}}_1$ as Down-Type Higgs

Choosing the curve $\overline{\mathbf{5}}_1$ as the down-type Higgs curve, we have the gauge invariant couplings

$$\overline{\mathbf{5}}_{H_d}\mathbf{10}_1\overline{\mathbf{5}}_6, \quad \overline{\mathbf{5}}_{H_d}\mathbf{10}_3\overline{\mathbf{5}}_3, \tag{A.1}$$

which lead to a rank-two down-type Yukawa matrix if the curves are not split and three generations come from two curves. We now ask whether the curves can be split in a way such that the rank is reduced to zero or one.

The relevant couplings in $\overline{\mathbf{5}}_{H_d}\mathbf{10}_M\overline{\mathbf{5}}_M$ in terms of SM representations are the ones which involve the Higgs doublet D:

$$D\bar{e}L, \quad D\bar{d}Q \tag{A.2}$$

The chiralities of the fields are given by the formulae (4.10),

$$\begin{aligned} n_{(3,1)_{-1/3}} - n_{(\bar{3},1)_{+1/3}} &= M_{\mathbf{5}} \\ n_{(1,2)_{+1/2}} - n_{(1,2)_{-1/2}} &= M_{\mathbf{5}} + N_Y \end{aligned} \tag{A.3}$$

for the $\mathbf{5}$ curves and (4.11) for the $\mathbf{10}$ curves:

$$\begin{aligned} n_{(3,2)_{+1/6}} - n_{(\bar{3},2)_{-1/6}} &= M_{\mathbf{10}} \\ n_{(\bar{3},1)_{-2/3}} - n_{(3,1)_{+2/3}} &= M_{\mathbf{10}} - N_Y \\ n_{(1,1)_{+1}} - n_{(1,1)_{-1}} &= M_{\mathbf{10}} + N_Y \end{aligned} \tag{A.4}$$

Our primary concern is the quark Yukawa matrix. Since the anti-down quark belongs to the representation $n_{(\bar{3},1)_{+1/3}}$ and the down quark belongs to $n_{(3,2)_{+1/6}}$, their chiralities are in our case fully determined by $M_{\mathbf{5}_3}$, $M_{\mathbf{5}_6}$ and $M_{\mathbf{10}_1}$ and $M_{\mathbf{10}_3}$. Overall, we need three generations from the $\overline{\mathbf{5}}$ matter curves and three generations from the $\mathbf{10}$ matter curves and therefore we have the conditions

$$M_{\mathbf{5}_3} + M_{\mathbf{5}_6} = -3, \quad M_{\mathbf{10}_1} + M_{\mathbf{10}_3} = 3. \tag{A.5}$$

If one sets one of the $M_{\mathbf{10}}$'s equal to one and the other one equal to two, we arrive at nothing new. Explicitly, choosing $M_{\mathbf{10}_1} = 1$ and $M_{\mathbf{10}_3} = 2$, one would demand that a heavy bottom quark is generated through the coupling $\overline{\mathbf{5}}_{H_d}\mathbf{10}_1\overline{\mathbf{5}}_6$ and thus set $M_{\mathbf{5}_6} = -1$ and $M_{\mathbf{5}_3} = -2$. But then the other term, $\overline{\mathbf{5}}_{H_d}\mathbf{10}_1\overline{\mathbf{5}}_6$, also exists and the matrix has rank two, which is exactly what we had before.

Thus, the only new option is $M_{\mathbf{10}_1} = 3$ and $M_{\mathbf{10}_3} = 0$ where the remaining relevant coupling is $\overline{\mathbf{5}}_{H_d}\mathbf{10}_1\overline{\mathbf{5}}_6$. $M_{\mathbf{5}_6} = 0$ then leads to a rank-zero matrix and all other values for $M_{\mathbf{5}_6}$ yield a rank-one matrix. In order not to introduce unnecessary fields, that is chiralities larger than three, one can see from the formulae for the other representations $n_{(\bar{3},1)_{-2/3}} - n_{(3,1)_{+2/3}} = M_{\mathbf{10}} - N_Y$ and $n_{(1,1)_{+1}} - n_{(1,1)_{-1}} = M_{\mathbf{10}} + N_Y$ that in the rank-one or -zero case it is in addition necessary that $N_{\mathbf{10}_1} = N_{\mathbf{10}_3} = 0$. Thus, this solution is a rather trivial one. It is important to note that nowhere in the above argumentation any use was made of the homology classes and the corresponding correlations between the different M's and N's as determined by the spectral cover approach.

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